ST PAUL'S SCHOOL
Est. 1509

## JUNIOR SCHOLARSHIP EXAMINATION

## MAY 2018

## MATHEMATICS

## 2 hours

Answer as many questions as you can in any order you wish. If you cannot do a question, move on to the next one and come back later.

Credit will be given for reasoning and working where appropriate.
When the answer is a fraction it should be given in mixed form, e.g. $3 \frac{4}{5}$
The total number of marks for this paper is 100 .
The mark allocation is shown in brackets at the end of each part of each question.

## CALCULATORS MAY NOT BE USED.

$\qquad$

1. Work out the values of:
(i) $0.03 \div 1.2$

## Answer

(ii) $\left(\frac{6}{13}\right)^{3}$

## Answer

(iii) $\frac{1}{10}-\frac{5}{14} \div 3 \frac{1}{8}$

Given that $3 a=2 b$, find the values of:
(i) $\frac{3 a^{2}}{2 b^{2}}$

## Answer

(ii) $\frac{\left(3 a^{2}\right)^{3}}{\left(8 b^{3}\right)^{2}}$

3 (i) The square of a prime number has how many factors?

> Answer
(ii) A particular positive whole number has exactly two factors. The next whole number has exactly three factors. What is the next whole number after that? Explain why this is the only solution.

Answer.

4 The triangle $A$ is reflected in the dotted line (which has equation $y=x+2$ ) to produce the triangle $B$. Triangle $B$ is rotated $90^{\circ}$ anticlockwise about the point $(5,3)$ to produce the triangle $C$.
Triangle $C$ is translated by the vector $\binom{1}{4}$ to produce the triangle $D$.
(i) Draw and label triangles $B, C$ and $D$ on the diagram.

(ii) Describe the single transformation that maps $A$ onto $D$.

Answer

5 A number 9 bus leaves every eight minutes and a number 10 bus leaves every nine minutes from Hammersmith bus station. A number 9 bus and a number 10 bus leave together at 8:00am.
(i) At what time will they next leave together?

Answer
(ii) Between 8:00am and 2:00pm, how many times do they leave exactly two minutes apart? Show your working.

6 Solve the following equations for $x$, giving answers as exact fractions where appropriate. Show all your working.
(i) $\frac{x+3}{2}-\frac{3 x-5}{3}=2$

## Answer

(ii) $17=12+\sqrt{\frac{x+3}{2 x-8}}$
(iii) Part (ii) is changed to $17=12-\sqrt{\frac{x+3}{2 x-8}}$.

Discuss.

## Answer

7 If all left-handed people in Herringskate are members of the local fishing club, which of the following statements are definitely true and which may not be true?
(i) All Herringskate fishing club members are left-handed.

> Answer
(ii) No Herringskate resident who is right-handed is a member of the local fishing club.

> Answer
(iii) If a left-handed person is not a member of the Herringskate fishing club, he or she does not live in Herringskate.

## Answer

8 (i) A pilot is setting out on a long scheduled flight but the plane is late to take off. The pilot estimates that he must cover the distance in $80 \%$ of the scheduled time. By what percentage must he increase his speed?

Answer
(ii) (a) By how much does one multiply a number to increase it by $16 \%$ ?

## Answer

$\qquad$
(b) A shopkeeper buys 30 game consoles, all at the same price. He sells 20 of them at a profit of $16 \%$ and has to sell the remaining 10 at a loss of $4 \%$. What is the overall percentage profit on the deal?

9 (i) Express as a single fraction:
(a) $p-\frac{1}{p}$
Answer ............................................. [1]
(b) $\frac{5}{2 p q}+p-\frac{1}{4 p}$

## Answer

(ii) Expand and simplify, showing all your steps:
$(3-2 y)(3+2 y)-(y-4)(2 y-3)$

10 (i) The rule for finding each subsequent term of a sequence is

$$
t(n+1)=2-\frac{2}{t(n)}
$$

where, if $t(n)$ is the current term, then $t(n+1)$ is the next term.
The first term is $t(1)=-1$. Using this rule, the second term is

$$
t(2)=2-\frac{2}{-1}=4
$$

(a) Find the next three terms using this rule.

## Answer

[3]
(b) Hence find the sum of the first 48 terms of this same sequence.

> Answer
(ii) If instead the first term is $t(1)=1$, what will happen in the sequence?
$\qquad$

11 Three digits 2, 5 and 8 are arranged at random to form a three-digit number. Find the probability that this number is a multiple of 4 .

## Answer

12 An equilateral triangle has sides of length 12 cm . Find its area.

## Answer

13 (i) Make r the subject of the formula $V=\frac{1}{3} \pi r^{2} h$.
(ii) Make $w$ the subject of the formula $v=\frac{1-2 w}{5 w-1}$

14 A square with sides of length 2 cm is rolled over three times in the same direction until the bottom left corner A is in the position of the bottom right corner.

Corner A takes the following path:

(i) What is distance between the initial position of A and the final position of A?

## Answer

(ii) What is the total length of the path that A takes from one side to the other?

15
In the diagram, $A B=A C, D E$ is at right angles to $A B$, angle $B D C$ is $92^{\circ}$ and angle $A B D$ is $42^{\circ}$. Calculate the size of angle $C B D$, providing two reasons.
(Diagram not drawn to scale)


## Answer

16 There is a large bag of red and blue counters. One red counter and one blue counter are removed from the bag. This process is repeated until $\frac{1}{4}$ of the red counters and $\frac{3}{5}$ of the blue counters have been removed. What fraction of the original counters remain in the bag?

17 A maths test consists of five questions. It is marked by giving 2 for each correct answer, -1 for each incorrect answer and 0 for each question which is not attempted.
(i) Write down the maximum and minimum marks for the whole test.

Answer
(ii) State the two possible ways in which a total of 0 can be scored for the test.

Answer
(iii) Show that just one of the totals between the maximum and the minimum is impossible to attain.

Answer.
(iv) If a candidate attempts all questions and is equally likely to be correct or wrong in each question, what is the probability that he or she will score exactly 7 ?

